CHAPTER 5

Gravitation

5.1 Introduction

By 1666, Newton had formulated and numerically checked the gravitation law he eventually published in his book *Principia* in 1687. Newton waited almost 20 years to publish his results because he could not justify his method of numerical calculation in which he considered Earth and the Moon as point masses. With mathematics formulated on calculus (which Newton later invented), we have a much easier time proving the problem Newton found so difficult in the seventeenth century.

Newton's law of universal gravitation states that *each mass particle attracts every other particle in the universe with a force that varies directly as the product of the two masses and inversely as the square of the distance between them.* In mathematical form, we write the law as

$$\mathbf{F} = -G\frac{mM}{r^2}\mathbf{e}_r \tag{5.1}$$

where at a distance r from a particle of mass M a second particle of mass m experiences an attractive force (see Figure 5-1). The unit vector $\mathbf{e}_r$ points from M to m, and the minus sign ensures that the force is attractive—that is, that m is attracted toward M.

A laboratory verification of the law and a determination of the value of G was made in 1798 by the English physicist Henry Cavendish (1731–1810). Cavendish's experiment, described in many elementary physics texts, used a torsion balance with two small spheres fixed at the ends of a light rod. The two spheres were attracted to two other large spheres that could be placed on either side of the smaller spheres. The official value for G is $6.673 \pm 0.010 \times 10^{-11}$ N · m^2/kg^2. Interestingly, although G is perhaps the oldest known of the fundamental constants,

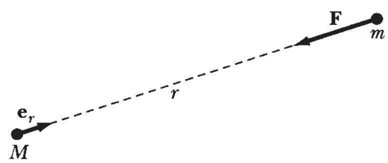

FIGURE 5-1 Particle m feels an attractive gravitational force toward M.

we know it with less precision than we know most of the modern fundamental constants such as e, c, and $\hbar$. Considerable research is ongoing today to improve the precision of G.

In the form of Equation 5.1, the law strictly applies only to *point particles*. If one or both of the particles is replaced by a body with a certain extension, we must make an additional hypothesis before we can calculate the force. We must assume that the gravitational force field is a *linear field*. In other words, we assume that it is possible to calculate the net gravitational force on a particle due to many other particles by simply taking the vector sum of all the individual forces. For a body consisting of a continuous distribution of matter, the sum becomes an integral (Figure 5-2):

$$\mathbf{F} = -Gm\int_V \frac{\rho(\mathbf{r}')\mathbf{e}_r}{r^2}dv' \tag{5.2}$$

where $\rho(\mathbf{r}')$ is the mass density and dv' is the element of volume at the position defined by the vector $\mathbf{r}'$ from the (arbitrary) origin to the point within the mass distribution.

If both the body of mass M and the body of mass m have finite extension, a second integration over the volume of m will be necessary to compute the total gravitational force.

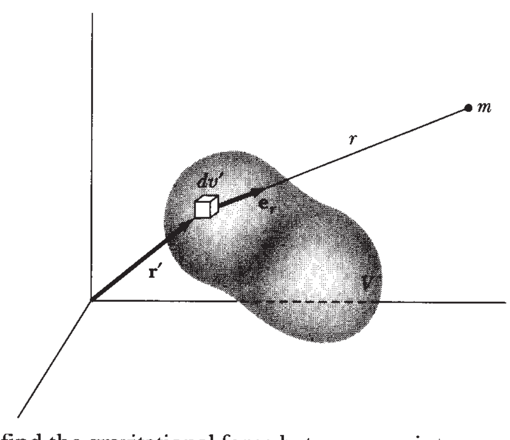

FIGURE 5-2 To find the gravitational force between a point mass m and a continuous distribution of matter, we integrate the mass density over the volume.

The **gravitational field vector g** is the vector representing the force per unit mass exerted on a particle in the field of a body of mass M. Thus

$$\mathbf{g} = \frac{\mathbf{F}}{m} = -G\frac{M}{r^2}\mathbf{e}_r \tag{5.3}$$

or

$$\boxed{\mathbf{g} = -G\int_V \frac{\rho(\mathbf{r}')\mathbf{e}_r}{r^2}dv'} \tag{5.4}$$

Note that the direction of $\mathbf{e}_r$ varies with r' (in Figure 5-2).

The quantity $\mathbf{g}$ has the dimensions of *force per unit mass*, also equal to *acceleration*. In fact, near the surface of the earth, the magnitude of $\mathbf{g}$ is just the quantity that we call the **gravitational acceleration constant.** Measurement with a simple pendulum (or some more sophisticated variation) is sufficient to show that $|\mathbf{g}|$ is approximately 9.80 m/s^2 (or 9.80 N/kg) at the surface of the earth.

5.2 Gravitational Potential

The gravitational field vector $\mathbf{g}$ varies as $1/r^2$ and therefore satisfies the requirement* that permits $\mathbf{g}$ to be represented as the gradient of a scalar function. Hence, we can write

$$\boxed{\mathbf{g} \equiv -\nabla\Phi} \tag{5.5}$$

where Φ is called the **gravitational potential** and has dimensions of (*force per unit mass*) × (*distance*), or *energy per unit mass*.

Because $\mathbf{g}$ has only a radial variation, the potential Φ can have at most a variation with r. Therefore, using Equation 5.3 for $\mathbf{g}$, we have

$$\nabla\Phi = \frac{d\Phi}{dr}\mathbf{e}_r = G\frac{M}{r^2}\mathbf{e}_r$$

Integrating, we obtain

$$\boxed{\Phi = -G\frac{M}{r}} \tag{5.6}$$

The possible constant of integration has been suppressed, because the potential is undetermined to within an additive constant; that is, only *differences* in potential are meaningful, not particular values. We usually remove the ambiguity in the value of the potential by arbitrarily requiring that $\Phi \rightarrow 0$ as $r \rightarrow \infty$; then Equation 5.6 correctly gives the potential for this condition.

*That is, $\nabla \times \mathbf{g} \equiv 0$.

The potential due to a continuous distribution of matter is

$$\Phi = -G\int_V \frac{\rho(\mathbf{r}')}{r}\,dv' \tag{5.7}$$

Similarly, if the mass is distributed only over a thin shell (i.e., a *surface* distribution), then

$$\Phi = -G\int_S \frac{\rho_s}{r}\,da' \tag{5.8}$$

where ρ_s is the surface density of mass (or *areal mass density*).

Finally, if there is a *line source* with linear mass density ρ_l, then

$$\Phi = -G\int_\Gamma \frac{\rho_l}{r}\,ds' \tag{5.9}$$

The physical significance of the gravitational potential function becomes clear if we consider the work per unit mass dW' that must be done by an outside agent on a body in a gravitational field to displace the body a distance $d\mathbf{r}$. In this case, work is equal to the scalar product of the force and the displacement. Thus, for the work done *on* the body per unit mass, we have

$$\begin{aligned} dW' &= -\mathbf{g}\cdot d\mathbf{r} = (\nabla\Phi)\cdot d\mathbf{r} \\ &= \sum_i \frac{\partial\Phi}{\partial x_i}\,dx_i = d\Phi \end{aligned} \tag{5.10}$$

because Φ is a function only of the coordinates of the point at which it is measured: $\Phi = \Phi(x_1, x_2, x_3) = \Phi(x_i)$. Therefore the amount of work per unit mass that must be done on a body to move it from one position to another in a gravitational field is equal to the difference in potential at the two points.

If the final position is farther from the source of mass M than the initial position, work has been done *on* the unit mass. The positions of the two points are arbitrary, and we may take one of them to be at infinity. If we define the potential to be zero at infinity, we may interpret Φ at any point to be the work per unit mass required to bring the body from infinity to that point. The *potential energy* is equal to the mass of the body multiplied by the potential Φ. If U is the potential energy, then

$$U = m\Phi \tag{5.11}$$

and the force on a body is given by the negative of the gradient of the potential energy of that body,

$$\mathbf{F} = -\nabla U \tag{5.12}$$

which is just the expression we have previously used (Equation 2.88).

We note that both the potential and the potential energy *increase* when work is done *on* the body. (The potential, according to our definition, is always negative and only approaches its maximum value, that is, zero, as r tends to infinity.)

A certain potential energy exists whenever a body is placed in the gravitational field of a source mass. This potential energy resides in the *field,** but it is customary under these circumstances to speak of the potential energy "of the body." We shall continue this practice here. We may also consider the source mass itself to have an intrinsic potential energy. This potential energy is equal to the gravitational energy released when the body was formed or, conversely, is equal to the energy that must be supplied (i.e., the work that must be done) to disperse the mass over the sphere at infinity. For example, when interstellar gas condenses to form a star, the gravitational energy released goes largely into the initial heating of the star. As the temperature increases, energy is radiated away as electromagnetic radiation. In all the problems we treat, the structure of the bodies is considered to remain unchanged during the process we are studying. Thus, there is no change in the intrinsic potential energy, and it may be neglected for the purposes of whatever calculation we are making.

EXAMPLE 5.1

What is the gravitational potential both inside and outside a spherical shell of inner radius b and outer radius a?

Solution. One of the important problems of gravitational theory concerns the calculation of the gravitational force due to a homogeneous sphere. This problem is a special case of the more general calculation for a homogeneous spherical shell. A solution to the problem of the shell can be obtained by directly computing the force on an arbitrary object of unit mass brought into the field (see Problem 5-6), but it is easier to use the potential method.

We consider the shell shown in Figure 5-3 and calculate the potential at point P a distance R from the center of the shell. Because the problem has symmetry about the line connecting the center of the sphere and the field point P, the azimuthal angle ϕ is not shown in Figure 5-3 and we can immediately integrate over $d\phi$ in the expression for the potential. Thus,

$$\begin{aligned}\Phi &= -G\int_V \frac{\rho(r')}{r}dv' \\ &= -2\pi\rho G\int_b^a r'^2 dr' \int_0^\pi \frac{\sin\theta}{r}d\theta\end{aligned} \tag{5.13}$$

where we have assumed a homogeneous mass distribution for the shell, $\rho(r') = \rho$. According to the law of cosines,

$$r^2 = r'^2 + R^2 - 2r'R\cos\theta \tag{5.14}$$

Because R is a constant, for a given r' we may differentiate this equation and obtain

$$2r\,dr = 2r'R\sin\theta\,d\theta$$

*See, however, the remarks at the end of Section 9.5 regarding the energy in a field.

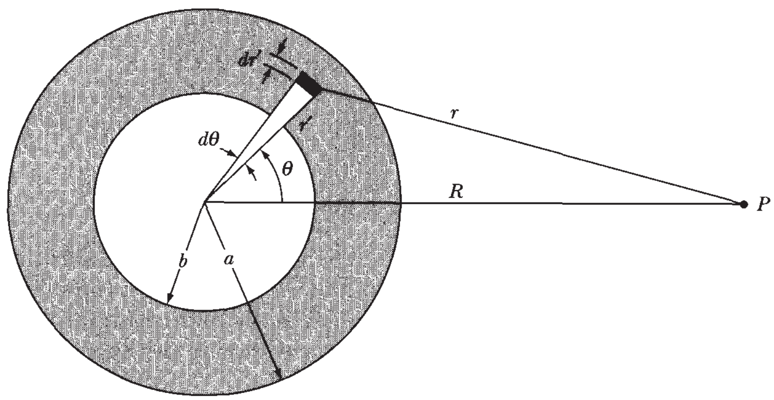

FIGURE 5-3 The geometry for finding the gravitational potential at point P due to a spherical shell of mass.

or

$$\frac{\sin\theta}{r}\,d\theta = \frac{dr}{r'R} \tag{5.15}$$

Substituting this expression into Equation 5.13, we have

$$\Phi = -\frac{2\pi\rho G}{R}\int_b^a r'\,dr'\int_{r_{\min}}^{r_{\max}} dr \tag{5.16}$$

The limits on the integral over dr depend on the location of point P. If P is *outside* the shell, then

$$\begin{aligned}\Phi(R > a) &= -\frac{2\pi\rho G}{R}\int_b^a r'\,dr'\int_{R-r'}^{R+r'} dr \\ &= -\frac{4\pi\rho G}{R}\int_b^a r'^2\,dr' \\ &= -\frac{4}{3}\frac{\pi\rho G}{R}(a^3 - b^3)\end{aligned} \tag{5.17}$$

But the mass M of the shell is

$$M = \frac{4}{3}\pi\rho(a^3 - b^3) \tag{5.18}$$

so the potential is

$$\boxed{\Phi(R > a) = -\frac{GM}{R}} \tag{5.19}$$

If the field point lies inside the shell, then

$$\begin{aligned}\Phi(R < b) &= -\frac{2\pi\rho G}{R}\int_b^a r'dr'\int_{r'-R}^{r'+R} dr \\ &= -4\pi\rho G\int_b^a r'dr' \\ &= -2\pi\rho G(a^2 - b^2) \end{aligned} \tag{5.20}$$

The potential is therefore constant and independent of position inside the shell.

Finally, if we wish to calculate the potential for points *within* the shell, we need only replace the lower limit of integration in the expression for $\Phi(R < b)$ by the variable R, replace the upper limit of integration in the expression for $\Phi(R > a)$ by R, and add the results. We find

$$\begin{aligned}\Phi(b < R < a) &= -\frac{4\pi\rho G}{3R}(R^3 - b^3) - 2\pi\rho G(a^2 - R^2) \\ &= -4\pi\rho G\left(\frac{a^2}{2} - \frac{b^3}{3R} - \frac{R^2}{6}\right) \end{aligned} \tag{5.21}$$

We see that if $R \rightarrow a$, then Equation 5.21 yields the same result as Equation 5.19 for the same limit. Similarly, Equations 5.21 and 5.20 produce the same result for the limit $R \rightarrow b$. The potential is therefore *continuous.* If the potential were not continuous at some point, the gradient of the potential—and hence, the force—would be infinite at that point. Because infinite forces do not represent physical reality, we conclude that realistic potential functions must always be continuous.

Note that we treated the mass shell as homogeneous. In order to perform calculations for a solid, massive body like a planet that has a spherically symmetric mass distribution, we could add up a number of shells or, if we choose, we could allow the density to change as a function of radius.

The results of Example 5.1 are very important. Equation 5.19 states that the potential at any point outside of a spherically symmetric distribution of matter (shell or solid, because solids are composed of many shells) is independent of the size of the distribution. Therefore, to calculate the external potential (or the force), we consider all the mass to be concentrated at the center. Equation 5.20 indicates that the potential is constant (and the force zero) anywhere inside a spherically symmetric mass shell. And finally, at points within the mass shell, the potential given by Equation 5.21 is consistent with both of the previous results.

The magnitude of the field vector $\mathbf{g}$ may be computed from $g = -d\Phi/dR$ for each of the three regions. The results are

$$\left.\begin{aligned} g(R < b) &= 0 \\ g(b < R < a) &= \frac{4\pi\rho G}{3}\left(\frac{b^3}{R^2} - R\right) \\ g(R > a) &= -\frac{GM}{R^2} \end{aligned}\right\} \tag{5.22}$$

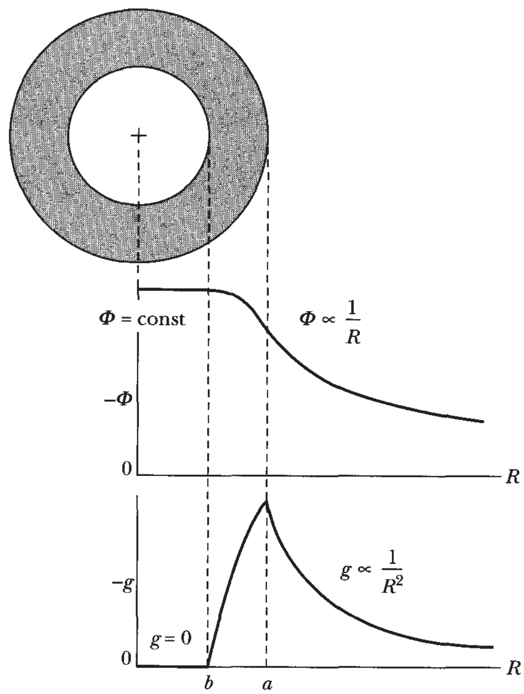

FIGURE 5-4 The results of Example 5.1 indicating the gravitational potential and magnitude of the field vector **g** (actually $-\mathbf{g}$) as a function of radial distance.

We see that not only the potential but also the field vector (and hence, the force) are continuous. The *derivative* of the field vector, however, is not continuous across the outer and inner surfaces of the shell.

All these results for the potential and the field vector can be summarized as in Figure 5-4.

EXAMPLE 5.2

Astronomical measurements indicate that the orbital speed of masses in many spiral galaxies rotating about their centers is approximately constant as a function of distance from the center of the galaxy (like our own Milky Way and our nearest neighbor Andromeda) as shown in Figure 5-5. Show that this experimental result is inconsistent with the galaxy having its mass concentrated near the center of the galaxy and can be explained if the mass of the galaxy increases with distance R.

Solution. We can find the expected orbital speed v due to the galaxy mass M that is within the radius R. In this case, however, the distance R may be hundreds of light years. We only assume the mass distribution is spherically symmetric. The gravitational force in this case is equal to the centripetal force due to the

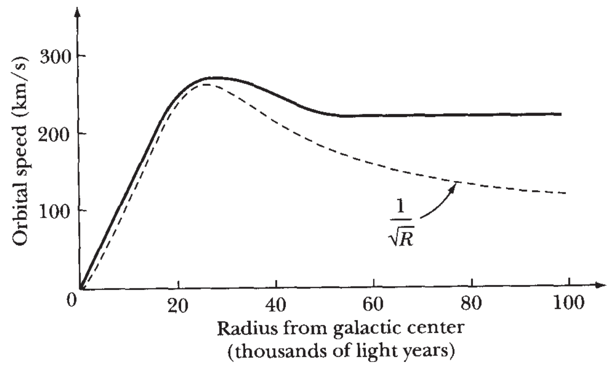

FIGURE 5-5 Example 5.2. The solid line represents data for the orbital speed of mass as a function of distance from the center of the Andromeda galaxy. The dashed line represents the $1/\sqrt{R}$ behavior expected from the Keplerian result of Newton's laws.

mass m having orbital speed v:

$$\frac{GMm}{r^2} = \frac{mv^2}{R}$$

We solve this equation for v:

$$v = \sqrt{\frac{GM}{R}}$$

If this were the case, we would expect the orbital speed to decrease as $1/\sqrt{R}$ as shown by the dashed line in Figure 5-5, whereas what is found experimentally is that v is constant as a function of R. This can only happen in the previous equation if the mass M of the galaxy itself is a linear function of R, $M(R) \propto R$. Astrophysicists conclude from this result that for many galaxies there must be matter other than that observed, and that this unobserved matter, often called "dark matter," must account for more than 90 percent of the known mass in the universe. This area of research is at the forefront of astrophysics today.

EXAMPLE 5.3

Consider a thin uniform circular ring of radius a and mass M. A mass m is placed in the plane of the ring. Find a position of equilibrium and determine whether it is stable.

Solution. From symmetry, we might believe that the mass m placed in the center of the ring (Figure 5-6) should be in equilibrium because it is uniformly surrounded by mass. Put mass m at a distance r' from the center of the ring, and place the x-axis along this direction.

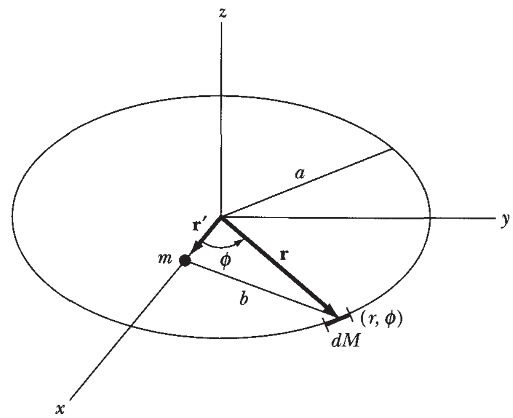

FIGURE 5-6 Example 5.3. The geometry of the point mass m and ring of mass M.

The potential is given by Equation 5.7 where $\rho = M/2\pi a$:

$$d\Phi = -G\frac{dM}{b} = -\frac{Ga\rho}{b}\,d\phi \tag{5.23}$$

where b is the distance between dM and m, and $dM = \rho a d\phi$. Let $\mathbf{r}$ and $\mathbf{r}'$ be the position vectors to dM and m, respectively.

$$\begin{aligned} b &= |\mathbf{r} - \mathbf{r}'| = |a\cos\phi\mathbf{e}_1 + a\sin\phi\mathbf{e}_2 - r'\mathbf{e}_1| \\ &= |(a\cos\phi - r')\mathbf{e}_1 + a\sin\phi\mathbf{e}_2| = [(a\cos\phi - r')^2 + a^2\sin^2\phi]^{1/2} \\ &= (a^2 + r'^2 - 2ar'\cos\phi)^{1/2} = a\left[1 + \left(\frac{r'}{a}\right)^2 - \frac{2r'}{a}\cos\phi\right]^{1/2} \end{aligned} \tag{5.24}$$

Integrating Equation 5.23 gives

$$\begin{aligned} \Phi(r') &= -G\int\frac{dM}{b} = -\rho aG\int_0^{2\pi}\frac{d\phi}{b} \\ &= -\rho G\int_0^{2\pi}\frac{d\phi}{\left[1 + \left(\frac{r'}{a}\right)^2 - \frac{2r'}{a}\cos\phi\right]^{1/2}} \end{aligned} \tag{5.25}$$

The integral in Equation 5.25 is difficult, so let us consider positions close to the equilibrium point, $r' = 0$. If $r' \ll a$, we can expand the denominator in Equation 5.25.

$$\begin{aligned} \left[1 + \left(\frac{r'}{a}\right)^2 - \frac{2r'}{a}\cos\phi\right]^{-1/2} &= 1 - \frac{1}{2}\left[\left(\frac{r'}{a}\right)^2 - \frac{2r'}{a}\cos\phi\right] \\ &\quad + \frac{3}{8}\left[\left(\frac{r'}{a}\right)^2 - \frac{2r'}{a}\cos\phi\right]^2 + \cdots \\ &= 1 + \frac{r'}{a}\cos\phi + \frac{1}{2}\left(\frac{r'}{a}\right)^2(3\cos^2\phi - 1) + \cdots \end{aligned} \tag{5.26}$$

Equation 5.25 becomes

$$\Phi(r') = -\rho G \int_0^{2\pi} \left\{ 1 + \frac{r'}{a}\cos\phi + \frac{1}{2}\left(\frac{r'}{a}\right)^2 (3\cos^2\phi - 1) + \cdots \right\} d\phi \quad \textbf{(5.27)}$$

which is easily integrated with the result

$$\Phi(r') = -\frac{MG}{a}\left[1 + \frac{1}{4}\left(\frac{r'}{a}\right)^2 + \cdots\right] \quad \textbf{(5.28)}$$

The potential energy $U(r')$ is from Equation 5.11, simply

$$U(r') = m\Phi(r') = -\frac{mMG}{a}\left[1 + \frac{1}{4}\left(\frac{r'}{a}\right)^2 + \cdots\right] \quad \textbf{(5.29)}$$

The position of equilibrium is found (from Equation 2.100) by

$$\frac{dU(r')}{dr'} = 0 = -\frac{mMG}{a}\frac{1}{2}\frac{r'}{a^2} + \cdots \quad \textbf{(5.30)}$$

so $r' = 0$ is an equilibrium point. We use Equation 2.103 to determine the stability:

$$\frac{d^2U(r')}{dr'^2} = -\frac{mMG}{2a^3} + \cdots < 0 \quad \textbf{(5.31)}$$

so the equilibrium point is unstable.

This last result is not obvious, because we might be led to believe that a small displacement from $r' = 0$ might still be returned to $r' = 0$ by the gravitational forces from all the mass in the ring surrounding it.

Poisson's Equation

It is useful to compare these properties of gravitational fields with some of the familiar results from electrostatics that were determined in the formulation of Maxwell's equations. Consider an arbitrary surface as in Figure 5-7 with a mass m placed somewhere inside. Similar to electric flux, let's find the gravitational flux Φ_m emanating from mass m through the arbitrary surface S.

$$\Phi_m = \int_S \mathbf{n}\cdot\mathbf{g}\, da \quad \textbf{(5.32)}$$

where the integral is over the surface S and the unit vector $\mathbf{n}$ is normal to the surface at the differential area da. If we substitute $\mathbf{g}$ from Equation 5.3 for

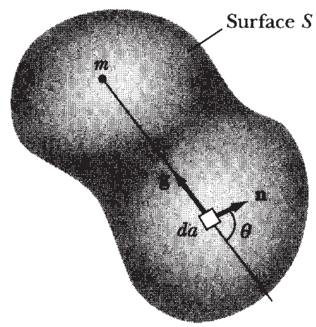

FIGURE 5-7 An arbitrary surface with a mass m placed inside. The unit vector $\mathbf{n}$ is normal to the surface at the differential area da.

the gravitational field vector for a body of mass m, we have for the scalar product $\mathbf{n} \cdot \mathbf{g}$,

$$\mathbf{n} \cdot \mathbf{g} = -Gm\frac{\cos\theta}{r^2}$$

where θ is the angle between $\mathbf{n}$ and $\mathbf{g}$. We substitute this into Equation 5.32 and obtain

$$\Phi_m = -Gm\int_S \frac{\cos\theta}{r^2}da$$

The integral is over the solid angle of the arbitrary surface and has the value 4π steradians, which gives for the mass flux

$$\Phi_m = \int_S \mathbf{n} \cdot \mathbf{g}\, da = -4\pi Gm \tag{5.33}$$

Note that it is immaterial where the mass is located inside the surface S. We can generalize this result for many masses m_i inside the surface S by summing over the masses.

$$\int_S \mathbf{n} \cdot \mathbf{g}\, da = -4\pi G\sum_i m_i \tag{5.34}$$

If we change to a continuous mass distribution within surface S, we have

$$\int_S \mathbf{n} \cdot \mathbf{g}\, da = -4\pi G\int_V \rho dv \tag{5.35}$$

where the integral on the right-hand side is over the volume V enclosed by S, ρ is the mass density, and dv is the differential volume. We use Gauss's divergence theorem to rewrite this result. Gauss's divergence theorem, Equation 1.130 where $d\mathbf{a} = \mathbf{n}\, da$, is

$$\int_S \mathbf{n} \cdot \mathbf{g}\, da = \int_V \nabla \cdot \mathbf{g}\, dv \tag{5.36}$$

If we set the right-hand sides of Equations 5.35 and 5.36 equal, we have

$$\int_V (-4\pi G)\rho dv = \int_V \nabla \cdot \mathbf{g} \, dv$$

and because the surface S, and its volume V, is completely arbitrary, the two integrands must be equal.

$$\nabla \cdot \mathbf{g} = -4\pi G\rho \tag{5.37}$$

This result is similar to the differential form of Gauss's law for electric field, $\nabla \cdot \mathbf{E} = \rho/\varepsilon$, where ρ in this case is the charge density.

We insert $\mathbf{g} = -\nabla\Phi$ from Equation 5.5 into the left-hand side of Equation 5.37 and obtain $\nabla \cdot \mathbf{g} = -\nabla \cdot \nabla\Phi = -\nabla^2\Phi$. Equation 5.37 becomes

$$\nabla^2\Phi = 4\pi G\rho \tag{5.38}$$

which is known as *Poisson's equation* and is useful in a number of potential theory applications. When the right-hand side of Equation 5.38 is zero, the result $\nabla^2\Phi = 0$ is an even better known equation called *Laplace's equation.* Poisson's equation is useful in developing Green's functions, whereas we often encounter Laplace's equation when dealing with various coordinate systems.

5.3 Lines of Force and Equipotential Surfaces

Let us consider a mass that gives rise to a gravitational field that can be described by a field vector $\mathbf{g}$. Let us draw a line outward from the surface of the mass such that the direction of the line at every point is the same as the direction of $\mathbf{g}$ at that point. This line will extend from the surface of the mass to infinity. Such a line is called a **line of force.**

By drawing similar lines from every small increment of surface area of the mass, we can indicate the direction of the force field at any arbitrary point in space. The lines of force for a single point mass are all straight lines extending from the mass to infinity. Defined in this way, the lines of force are related only to the *direction* of the force field at any point. We may consider, however, that the *density* of such lines—that is, the number of lines passing through a unit area oriented perpendicular to the lines—is proportional to the *magnitude* of the force at that area. The lines-of-force picture is thus a convenient way to visualize both the magnitude and the direction (i.e., the *vector* property) of the field.

The potential function is defined at every point in space (except at the position of a point mass). Therefore, the equation

$$\Phi = \Phi(x_1, x_2, x_3) = \text{constant} \tag{5.39}$$

defines a surface on which the potential is constant. Such a surface is called an **equipotential surface.** The field vector $\mathbf{g}$ is equal to the gradient of Φ, so $\mathbf{g}$ can

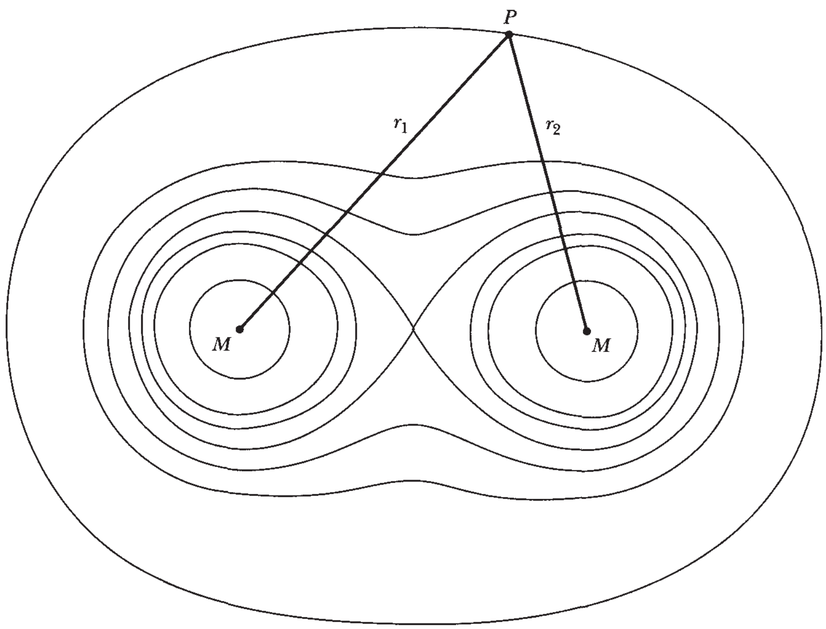

FIGURE 5-8 The equipotential surfaces due to two point masses M.

have no component *along* an equipotential surface. It therefore follows that every line of force must be normal to every equipotential surface. Thus, the field does no work on a body moving along an equipotential surface. Because the potential function is single valued, no two equipotential surfaces can intersect or touch. The surfaces of equal potential that surround a single, isolated point mass (or any spherically symmetric mass) are all spheres. Consider two point masses M that are separated by a certain distance. If r_1 is the distance from one mass to some point in space and if r_2 is the distance from the other mass to the same point, then

$$\Phi = -GM\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \text{constant} \tag{5.40}$$

defines the equipotential surfaces. Several of these surfaces are shown in Figure 5-8 for this two-particle system. In three dimensions, the surfaces are generated by rotating this diagram around the line connecting the two masses.

5.4 When Is the Potential Concept Useful?

The use of potentials to describe the effects of "action-at-a-distance" forces is an extremely important and powerful technique. We should not, however, lose sight of the fact that the ultimate justification for using a potential is to provide a